On $\mathbb{C}$ define $\sim$: $z_1 \sim z_2 \Leftrightarrow |z_1| = |z_2|$

$\sim$: is an equivalance relation on $\mathbb{C}$

$\hat{0}$, $\widehat{1+3i}$, $\widehat{a+bi}$ = ?

System of representatives

Let $f: \mathbb{C} \to \mathbb{R}$ Then $\sim = \sim_f \Rightarrow \sim$ is an equivalance relation on $\mathbb{C}$

$f(z) = |z|$

$z \mapsto |z|$

$x \sim 0 \Leftrightarrow |x| = |0| = 0$

Let $x = a+bi$; $a, b \in \mathbb{R}$. Then $|x| = \sqrt{a^2+b^2} \Rightarrow$

$\Rightarrow \sqrt{a^2+b^2} = 0 \Rightarrow a^2 + b^2 = 0$

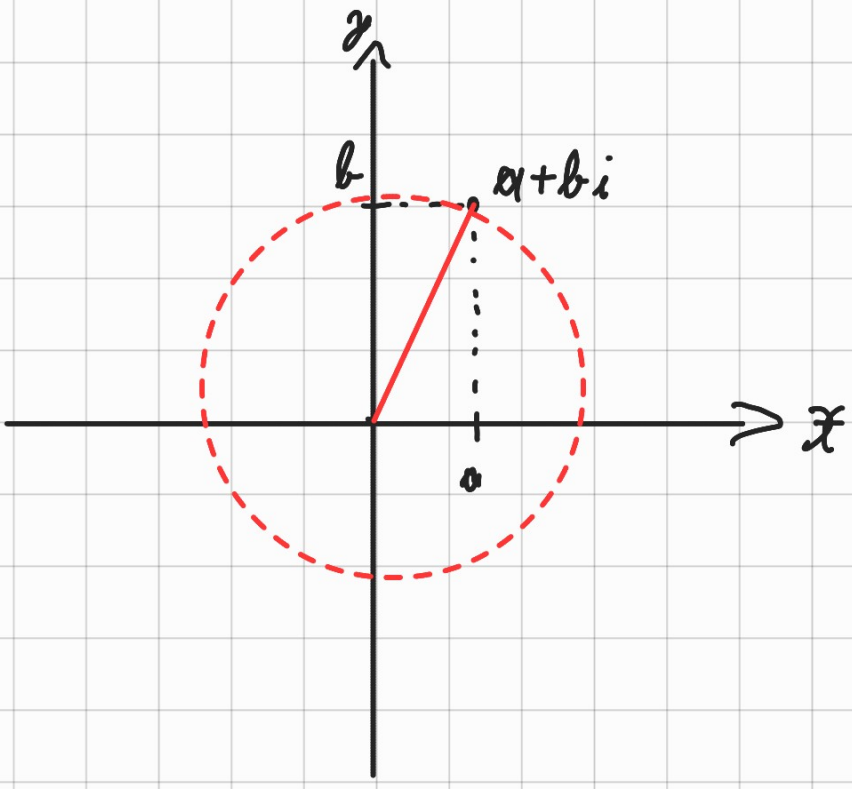
$a, b \in \mathbb{R} \Rightarrow \begin{cases} a^2 \geq 0 \\ b^2 \geq 0 \end{cases} \Rightarrow a = b = 0 \Rightarrow x = 0$

$x \sim (a+ib) \Leftrightarrow |x| = |a+bi| \Leftrightarrow |x| = \sqrt{a^2+b^2} \Rightarrow$

Let $x = c+di$, then $|x| = \sqrt{c^2+d^2}$

$c, d \in \mathbb{R}$

$\Rightarrow \sqrt{c^2+d^2} = \sqrt{a^2+b^2} \Leftrightarrow c^2+d^2 = a^2+b^2$



$\widehat{a+bi} = \mathcal{C}(O, \sqrt{a^2+b^2})$

The trigonometric form of $z \in \mathbb{C}^*$

$z = |z| \cdot (\cos\alpha + i \sin\alpha)$ α unique in $[0, 2\pi)$

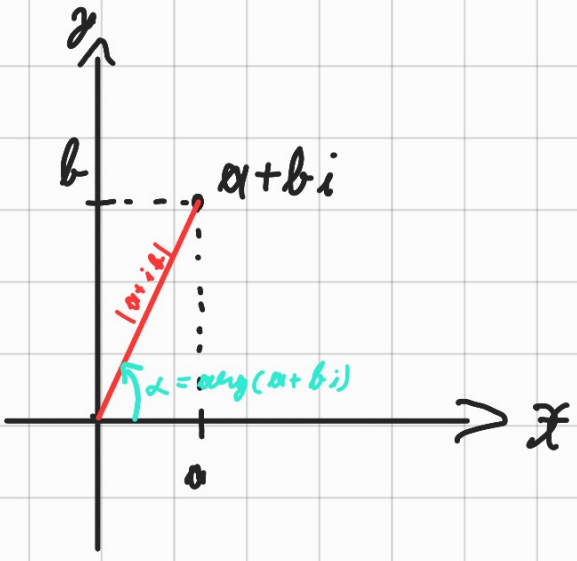
$\alpha = \arg z$ „the reduce argument of z"

On $\mathbb{C}$ define $\sim$: $z_1 \sim z_2 \Leftrightarrow \arg z_1 = \arg z_2$

$\sim$ is an equivalance relation on $\mathbb{C}$

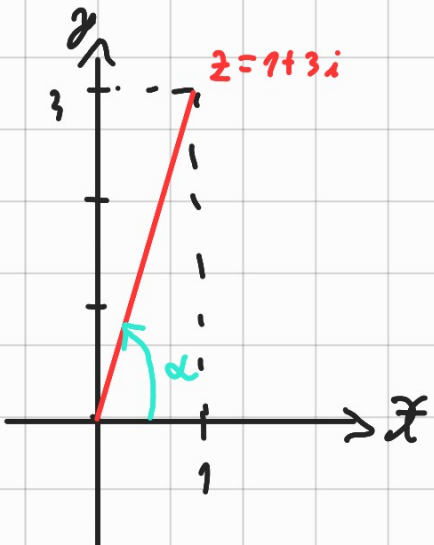
$\widehat{1+3i}$, $\widehat{a+bi}$ = ?

System of representatives



Let $f: \mathbb{C}^* \to \mathbb{R}$. $f(z) = \arg z$

Then $\sim = \sim_f \Rightarrow \sim$ is an equivalence relation on $\mathbb{C}^*$



$\operatorname{tg}\alpha = \frac{3}{1} = 3$

$\alpha = \operatorname{arctg}(3)$

$\widehat{1+3i} = \{x+yi \in \mathbb{C}^* \mid \arg(x+iy) = \operatorname{arctg}(3)\}$

$\widehat{a+bi}$ = the half line from 0 through $a+bi$
the ray

$\mathcal{C}(0,1)$ is a system of representatives

On $\mathbb{R} \times \mathbb{R}$ define $\sim$

$(x_1,y_1) \sim (x_2,y_2) \Leftrightarrow 2x_1 + 5y_1 + 3 = 2x_2 + 5y_2 + 3$

1) $\sim$ is an equivalance rel. on $\mathbb{R}\times\mathbb{R}$

2) Find $(\widehat{0,1})$, $(\widehat{2,3})$, $(\widehat{a,b})$

3) Find a system of representatives for η

Let $f: \mathbb{R}\times\mathbb{R} \to \mathbb{R}$; $f(x,y) = 2x+5y+3$

then $\sim = \sim_f \Rightarrow \sim$ is an equivalance relation on $\mathbb{R}\times\mathbb{R}$

$(x,y) \sim (0,1) \Rightarrow 2x+5y+3 = 2\cdot 0 + 5\cdot 1 + 3 \iff$

$\iff 2x+5y = 5$

$\iff x = \frac{5-5y}{2}$

$\widehat{(0,1)} = \left\{ \left(\frac{5-5y}{2}, y \right) \mid y \in \mathbb{R} \right\}$

OR $\quad 2x+5y = 5 \iff y = \frac{5-2x}{5}$

$\widehat{(0,1)} = \left\{ \left(x, \frac{5-2x}{5} \right) \mid x \in \mathbb{R} \right\}$

$(x,y) \sim (a,b) \Rightarrow 2x+5y+3 = 2a+5b+3$

$\iff 2x+5y = 2a+5b$

$\iff 2x = 2a+5b-5y$

$\iff x = \frac{2a+5b-5y}{2}$

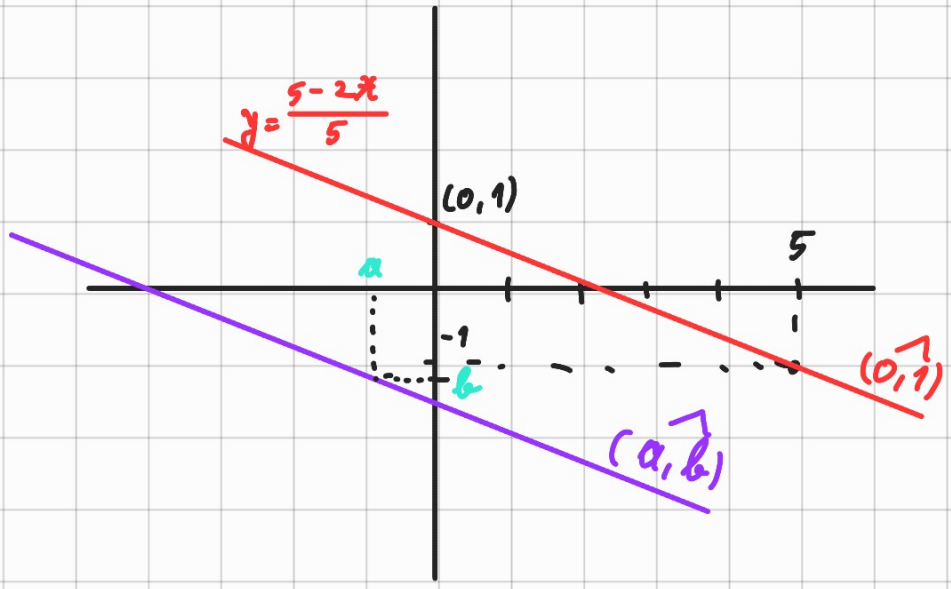
$\widehat{(a,b)} = \left\{ \left(\frac{2a+5b-5y}{2}, y \right) \mid y \in \mathbb{R} \right\}$

$$\text{OR} \quad 2x+5y = 2a+5b$$

$$\Leftrightarrow 5y = 2a+5b$$

$$\Leftrightarrow y = \frac{2a+5b-5y}{5}$$

$$\overrightarrow{(a,b)} = \left\{ \left(x, \frac{2a+5b-5y}{5} \right) \;\middle|\; x \in \mathbb{R} \right\}$$



Divisibility in $\mathbb{Z}$

$x, y \in \mathbb{Z}$ $\quad x \mid y$ if $\exists\, k \in \mathbb{Z}$ $\quad y = k \cdot x$

$(\mathbb{N}, \mid)$ is an ordered set:

Reflexivity: $\quad x \in \mathbb{N} \qquad x \mid x$

Transitive: $\quad x \mid y$ and $y \mid z \Rightarrow \begin{matrix} y = k_1 \cdot x \\ z = k_2 \cdot y \end{matrix} \Big\} \Leftrightarrow$

$$\Leftrightarrow z = \underbrace{(k_1 \cdot k_2)}_{\in \mathbb{N}}\, x \Rightarrow x \mid z$$

Antisymmetry: Let $x, y \in \mathbb{N}$ $\quad x \mid y$ and $y \mid x$

$$\Rightarrow \begin{cases} y = x \cdot k_1, & k_1 \in \mathbb{Z} \\ x = y \cdot k_2, & k_2 \in \mathbb{Z} \end{cases} \Rightarrow x = (x \cdot k_1) \cdot k_2 = x \cdot k_1 \cdot k_2 \Rightarrow$$

$$\Rightarrow \begin{cases} x \neq 0: \; k_1 \cdot k_2 = 1 \Rightarrow k_1 = k_2 = 1 \Rightarrow x = y \\ x = 0 \Rightarrow y = x \cdot k_1 = 0 \end{cases} \Big\} \Rightarrow x = y$$

$\Rightarrow$ „$\mid$" is an order relation on $\mathbb{N}$ divisibility

$PD_n = D_n \setminus \{1, n\}$ the proper divisors of n

For $n \in \mathbb{N}^*$ define D_n = the set of divisors of n in $\mathbb{N}$

$(D_n, |)$ is an ordered set

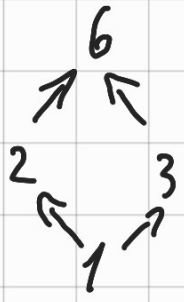
Draw the Hasse diagram for $(D_n, |)$

when $n = 4, 6, 12, 16, 25, 100$

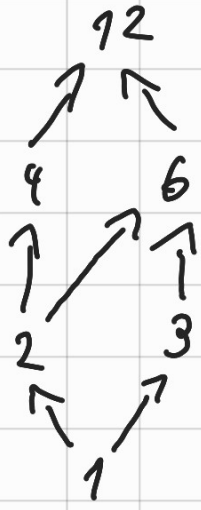
$D_4 = \{1, 2, 4\}$



$D_6 = \{1, 2, 3, 6\}$



$D_{12} = \{1, 2, 3, 4, 6, 12\}$



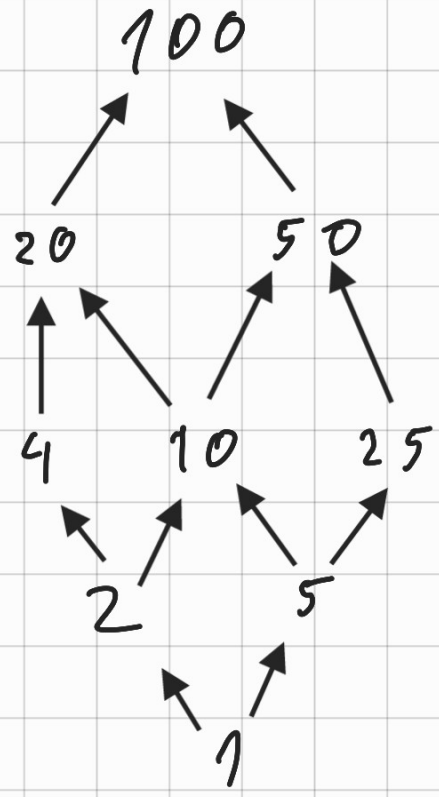
$D_{16} = \{1, 2, 4, 8, 16\}$



$D_{25} = \{1, 5, 25\}$



$D_{100} = \{1, 2, 4, , 5, 10, 20, 25, 50, 100\}$



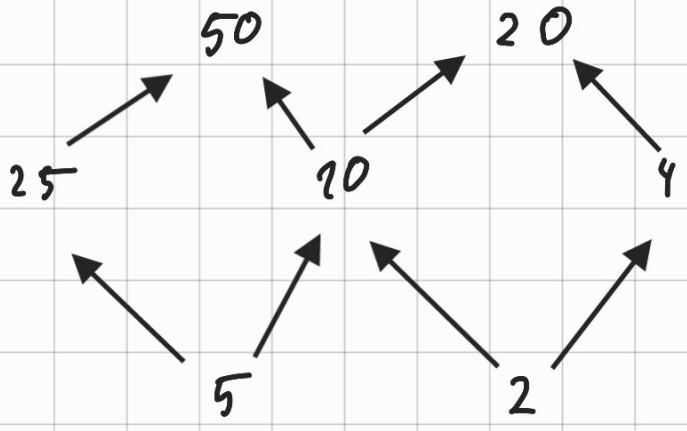
$\forall n \in (\mathbb{N}, 1)$ $a \mid b \iff \frac{b}{a}$ is prime

$D_n = ?$ for $n = 30, 60$

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$PD(25)$: 5

$PD(100)$:



5, 2 are minimal elements in $(PD(100), |)$

50, 20 are maximal elements in $(PD(100), |)$

