Relations

Let A, B sets. A relation from A to B is an subset $\rho \subseteq A \times B$

Defn: $\rho, \eta, R, \sim$ for relations

The inverse of $\rho \subseteq A \times B$ is

$$\rho^{-1} := \{(y, x) \in B \times A \mid (x, y) \in \rho\} \subseteq B \times A$$

- If $\rho_1 \subseteq A \times B$ and $\rho_2 \subseteq B \times C$, then

composition is:

$$\rho_1 \circ \rho_2 = \{(x, z) \in A \times C \mid \exists\, y \in B \quad (x, y) \in \rho_1, \ (y, z) \in \rho_2\}$$

Special case: if $A = B$, then $\rho \subseteq A \times A$ is called a (binary) relation on A

Example: Given $A = \phi$, denote

$\Delta_A = \{(x, x) \mid x \in A\} \subseteq A \times A$ is called

"the diagonal relation"

Defn : Let $A \neq \phi$ and the relation ρ is called :

1) Reflexive if $\forall x \in A \quad (x,x) \in \rho \Leftrightarrow \Delta_A \subseteq \rho$

2) Symmetric if $\forall x,y \in A \quad (x,y) \in \rho$ and $(y,x) \in \rho \Rightarrow$
$\Leftrightarrow x = y \Leftrightarrow \rho^{-1} = \rho$

3) Antisymmetric if $\forall x,y \in A \quad (x,y) \in \rho$ and $(y,z) \in \rho$
$\Rightarrow (x,z) \in \rho$

4) Transitive if $\forall x,y,z \in A$

$[\,(x,y) \in \rho$ and $(y,z) \in \rho \Rightarrow (x,z) \in \rho\,] \Leftrightarrow$

$\Leftrightarrow \rho \circ \rho \subseteq \rho$

Not: $(x,y) \in \rho \hookrightarrow x \;\rho\; y$

$(x,y) \notin \rho \hookrightarrow x \not\rho\; y$

Notes:

$$\rho \circ \rho = \{ (x,z) \mid \exists\, y \in A \quad \begin{matrix} (x,y) \in \rho \\ (y,z) \in \rho \end{matrix} \text{ and } \}$$