Index sets

Say $M \neq \emptyset$

A sequence of elements in M is a function

$x : \mathbb{N} \to M$

$\forall\, n \in \mathbb{N} \longmapsto x(n) \in M$

$x(n)$ denote x_n

$(x_n)_{n \in \mathbb{N}}$ Here $\mathbb{N}$ is the set of indices

For $I \neq \emptyset$ a set of indices. A family of elements in M indexed by I is a function

$x : I \to M$

$\forall\, i \in I \rightsquigarrow x(i) \in M$

Shortly: $(x_i)_{i \in I}$

I family of indices

Indexed family of sets: $(A_i)_{i \in I}$

Operations

- Union: $\bigcup_{i \in I} A_i = \{x \mid \exists\, i \in I \text{ s.t. } x \in A_i\}$

- Intersection: $\bigcap_{i \in I} A_i = \{x \mid x \in A_i,\ \forall\, i \in I\}$

- Distributivity: $\left(\bigcup_{i \in I} A_i\right) \cap B = \bigcup_{i \in I} (A_i \cap B)$

$$\left(\bigcup_{i \in I} A_i\right) \cap \left(\bigcup_{j \in J} B_j\right) = \bigcup_{\substack{i \in I \\ j \in J}} (A_i \cap B_j)$$

$$\left(\bigcap_{i \in I} A_i\right) \cup \left(\bigcap_{j \in J} B_j\right) = \bigcap_{(i,j) \in I \times J} (A_i \cap B_j)$$

- Cartesian product: $\prod_{i \in I} A_i = \{ x : I \to \bigcup_{x \in I} A_i \mid x(i) \in A_i \ \forall i \in I \}$ of an indexed family of sets

These are called selection functions for the family $(A_i)_{i \in I}$

The axiom of choice

If $I \neq \phi$ and $A_i \neq \phi \ \forall i \in \forall_i \Rightarrow \prod_{i \in I} A_i \neq \phi$